MOST LIKELY QUESTIONS (2022 EXAM)

RELATIONS AND FUNCTIONS

$(1M \times 2 + 2M \times 1 + 3M \times 2 + 5M \times 2 = 17(7)$

ONE MARK QUESTIONS

1. Define Reflexive, Symmetric, Transitive and Equivalence relations.
2. A relation R in the set {1,2,3} given that $R = \{(1,2), (2,1), (1,1)\}$ is not transitive.Why ?
3. A relation R in the set {1,2,3} given by $R = \{(1,1), (2,2), (3,3), (1,2), (2,3)\}$ is not symmetric. Why ?
4. Give an example of a relation for the following
   a) Reflexive bur neither symmetric nor transitive
   b) Symmetric but neither reflexive nor transitive
   c) Transitive only
   d) Reflexive and symmetric bur not transitive
   e) Reflexive and transitive but not symmetric.
   f) Symmetric and transitive but not reflexive
   g) Reflexive, symmetric and transitive.
5. Find the number of all one-one functions from set A = {1, 2, 3} to itself.
6. Define bijective function.
7. Define binary operation.
8. Verify $*$ is a binary operation on the following signs.
   1. $a * b = a + b$ on N
   2. $a * b = a - b$ on N
   3. $a * b = a - b$ on Z
   4. $a * b = a - b$ on R
   5. $a * b = \frac{a-b}{a+b}$ on Z
   6. $a * b = 3a - 2b$ on N
9. If $*$ is defined on the set Nas $a * b = L.C.M. of\ a \,\&\, b$.Find 5*7
10. If $*$ is defined on the set of N as $a * b = L.C.M.$ of $a \,\&\, b$.Find 16*20.

TWO MARK QUESTIONS

1. Find gof and fog, if $f : R \to R$ and $g : R \to R$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$
2. Find gof and fog, if $f(x) = 8x^3$ and$(x) = x^{\frac{1}{3}}$.
3. Find gof and fog, if $f(x) = |x|$ and $g(x) = |5x - 2|$

4. If $f: R \to R$ defined by $f(x) = (3 - x^3)^{\frac{1}{3}}$ then prove that $fof\ (x) = x$.
5. If $f: A \to B$ and $g: B \to C$ are one-one then prove that $gof: A \to C$ is also one-one.
6. If $*$ is binary operation on the set of rational numbers defined as follows then find which of the following binary operations are commutative or associative.

   1) $a * b = a - b$ 2) $a * b = a + ab$ 3) $a * b = ab + 1$ 4) $a * b = \frac{ab}{4}$

   5) $a * b = \frac{3ab}{7}$.

THREE MARK QUESTIONS

1. Let T be the set of all triangles in a plane with R a relation in T given by R = {(T1, T2) : T1 is congruent to T2}. Show that R is an equivalence relation.
2. Show that the relation R in the set {1, 2, 3} given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.
3. Show that the relation R in the set **Z** of integers given by R = {(a, b) : 2 divides $a - b$} is an equivalence relation.
4. Show that relation R in the set A = {1, 2, 3, ..., 13, 14} defined as R = {(x, y) : $3x - y = 0$} is an equivalence relation.
5. Determine whether the Relation R in the set $\mathbb{Z}$ of all integers defined as $R = \{(x, y): x - y$ is an integer} is reflexive, symmetric & transitive:
6. Check whether the relation R in the set **N** of natural numbers defined as R = {(x, y) : $y = x + 5$ and $x < 4$} is reflexive, symmetric & transitive:
7. Show that the relation R in in the set A = {1, 2, 3, 4, 5, 6} as R = {(x, y) : y is divisible by x} is an equivalence relation.
8. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : $|a - b|$ is even}, is an equivalence relation.
9. If $R = \{(a, b) | a - b \text{ is a multiple of } 4\}$ is an equivalence relation in the set $A = \{0,1,2, ...9\}$.
10. Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2): T_1 \text{ is similar to } T_2\}$ is equivalence relation.?
11. Let L be the set of all lines in xy plane and R be the relation in L defined as R={(L_1,L_2) : L_1 is parallel to L_2}. Show that R is an Equivalence relation.
12. Determine whether the operation * defined by $a * b = \frac{ab}{7}$ is associative and commutative on the set Q_0 of all nonzero rational numbers .

13. On Q, define $a * b = ab + 1$.Determine whether $*$ is binary, commutative or associative.

14. On Q, define $a * b = \frac{ab}{2}$. Determine whether $*$ is binary, commutative or associative

15. Let $*$ be a binary operation on the setQ of rational numbers defined as $a * b = a + ab$. Is $*$ commutative? Is $*$ associative?

FIVE MARKS MARK QUESTIONS

1. Let $f: R \to R$ be given by $(x) = 4x + 3$. Show that f is invertible and find the inverse of f.
2. Consider $f: R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. S.T f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y - 4}$ where R_+ is the set of all non-negative real numbers.
3. Let $f: N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$ for some $x$in N, show that $f: N \to S$, where S is the range of f, is invertible. Find the inverse of f.
4. Consider $f: R_+ \to [-5, \infty)$ $given\ by$ $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$.
5. Let Y = $\{n^2 : n \in N\} \subset N$. Consider f : N → Y as $f(n) = n^2$. Show that f is invertible. Find the inverse of f.
6. Show that the function $f : R_* \to R_*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto, where R_* is the set of all non-zero real numbers.
7. If A=R-{3} and B=R-{1} and f:A→B is a function defined by $f(x) = \frac{x-2}{x-3}$. Is f $one - one$ and onto? Justify your answer.
8. Prove that the function f : R → R, given by $f(x) = 2x$, is one-one and onto.
9. Show that the function f : R → R, defined as $f(x) = x^2$, is neither one-one nor onto.
10. State whether the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3 - 4x$ is one-one, onto or bijective.
11. State whether the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 1 + x^2$ is one-one, onto or bijective.

INVERSE TRIGONOMETRIC FUNCTIONS

$(\mathbf{1M \times 2 + 2M \times 2 + 3M \times 1 = 09(5)})$

ONE MARK QUESTIONS

1.Write the domain and range of inverse trigonometric functions.

2. Find the principal value of the following :

1) $\sin^{-1}\frac{1}{\sqrt{2}}$ 2) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ 3) $cosec^{-1}2$ 4) $\tan^{-1}(-\sqrt{3})$

5) $\tan^{-1}(-1)$ 6) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ 7) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ 8) $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

3. Find the value of $\cot(\tan^{-1}x + \cot^{-1}x)$.
4. Find the value of $\cos(\sec^{-1}x + cosec^{-1}x)$, $|x| \geq 1$
5. For what value of x the following functions Satisfies

1) $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x$

2) $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x$

3) $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$

6.If $\sin^{-1}x = y$, then find the set of value of y.

TWO MARK QUESTIONS

1. Prove that $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$
2. Prove that $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$.
3. Prove that $\cos^{-1}(-x) = \pi - \cos^{-1}x$
4. Prove that $\sin^{-1}(-x) = -\sin^{-1}x$
5. Find the value of the following

1) $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

2) $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

3) $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

4) $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$

5) $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$

6) $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

6. Find the value of the following:

1) $\sin^{-1}\sin\left(\frac{2\pi}{3}\right)$ 2) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ 3) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ 4) $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

7. If $\tan^{-1}x = \frac{\pi}{10}$ then find $\cot^{-1}x$
8. Prove the following

   1) $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x\ , |x| \le 1$

   2) $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x\ , x \ge 0$

   3) $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x\ , |x| < 1$

   4) $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1}x\ , -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

   5) $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\cos^{-1}x\ , \frac{1}{\sqrt{2}} \le x \le 1$

THREE MARKS QUESTIONS

1. Write the following functions in the simplest form:

   1) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \neq 0$ 2) $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)\ , -\frac{a}{\sqrt{3}} < x < \frac{a}{\sqrt{3}}$

   3) $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)\ , |x| < a$ 4) $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)\ , x < \pi$

   5) $\tan^{-1}\left[\frac{2\cos x-3\sin x}{3\cos x+2\sin x}\right],\ where\ \frac{2}{3}\tan x > -1.$

   6) $\cot^{-1}\frac{1}{\sqrt{x^2-1}}, |x| > 1$ 7) $\tan^{-1}\frac{1}{\sqrt{x^2-1}}\ , |x| > 1$

2. Prove the following:

   1) $\tan^{-1}\left[\frac{1+sinx}{cosx}\right] = \frac{\pi}{4}+\frac{x}{2}$ 2) $\tan^{-1}\left(\frac{cosx}{1-sinx}\right) = \frac{\pi}{4}+\frac{x}{2}$

   3) $\tan^{-1}\left[\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right] = \tan^{-1}\frac{a}{b}-x$

   4) $2\tan^{-1}\frac{1}{2}+\ \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

   5) $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{2}{11}\right) = \tan^{-1}\left(\frac{4}{3}\right).$

   6) $\tan^{-1}x+\ \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

3. If $\tan^{-1}2\text{x}+\tan^{-1}3\text{x} = \frac{\pi}{4}$ then find x
4. If $\tan^{-1}\left(\frac{\text{x}-1}{\text{x}-2}\right)+\tan^{-1}\left(\frac{\text{x}+1}{\text{x}+2}\right) = \frac{\pi}{4}$ then find x.

5) $2\tan^{-1}(\cos x) = \tan^{-1}(2\ cosecx\ cotx)$

9. Write the following functions in the simplest form:
10. Prove that $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

11. Prove that $2\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{24}{7}\right)$.

12. If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, then find the value of x.

13. Find x if $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$

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MATRICES

$(1M \times 1 + 2M \times 1 + 3M \times 1 + 5M \times 1 = 11(4))$

ONE MARK QUESTIONS

1. Construct a 2×2 matrix , $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{(i+j)^2}{2}$
2. Construct a 2×2 matrix , $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{(i+2j)^2}{2}$
3. Construct a 3×2 matrix, $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{1}{2}|i - 3j|$
4. Construct a 2 X 2matrix $A = [a_{ij}]$. Whose element are given by $a_{ij} = 2i + j$.
5. Construct a 2 X 2matrix $A = [a_{ij}]$. Whose element are given by $a_{ij} = \frac{i}{j}$.
6. If the matrix has 24 elements what are the possible order it can have?
7. If a matrix has 8 elements, what are the possible orders it can have?
8. Find the number of all possible matrices of order 3×3 with each entry 0 or 1.

1) Definitions: 1) Square matrix 2)Diagonal matrix 3)Scalar matrix 4)Symmetric Matrix 5)Skew-Symmetric Matrix.

TWO MARK QUESTIONS

1. If $A = \begin{bmatrix} \sqrt{3} & 1 & -1 \\ 2 & 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & \sqrt{5} & 1 \\ -2 & 3 & \frac{1}{2} \end{bmatrix}$ then find $A + B$
2. Find X, if $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$.
3. Find Y, if $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.
4. If $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$. Show that $AB = O$.
5. $If\ 2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ then find x and y.
6. If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ then find AB
7. Find the value of $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

8. If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew-symmetric then find a, b and c.

9. If $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ then find a, b, c and d.

10. If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, then find AB, BA. Show that $AB \neq BA$.

11. If $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$, then find the matrix X, such that2A+3X=5B.

THREE MARKS QUESTIONS

1. For any square matrix A with Real numbers. Prove that $\mathrm{A} + \mathrm{A}'$ is a symmetric and $\mathrm{A} - \mathrm{A}'$ is a skew symmetric.
2. Any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix.
3. If A and B are symmetric matrices of same order then prove that AB is symmetric if and only if $AB = BA$.
4. Express the following matrices as the sum of symmetric and skew symmetric matrix:

   i) $\begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix}$ ii) $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

4. For the matrix $\mathrm{A} = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

   (i) $\mathrm{A} + \mathrm{A}'$ is a symmetric matrix.

   (ii) $A - A'$ is a skew- symmetric matrix.

5. Prove that Inverse of a square matrix, if it exists, is unique.
6. Prove that if A and B are invertible matrix of same order then $(AB)^{-1} = B^{-1}A^{-1}$.
7. If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix.
8. Find the inverse of the following matrices using elementary operation

   i) $\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ ii) $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ iii) $\begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$ iv) $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

FIVE MARKS QUESTIONS

1. If $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{pmatrix}$ *and* $C = \begin{pmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{pmatrix}$ then prove that $A(BC) = (AB)C$

2. If $A = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, verify that $(A+B)' = A' + B'$.

3. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then show that $(A-B)' = A' - B'$.

4. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$, *and* $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$. Calculate AC, BC *and* $(A+B)C$. Also, verify that $(A+B)C = AC + BC$.

5. . If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$ then compute $A+B$ and $B-C$. Also, verify that $A+(B-C) = (A+B)-C$.

6. If $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$, verify that $(AB)' = B'A'$

7. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}$ Verify that $(AB)' = B'A'$.

8. If A= $\begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}$ then prove that $A^3 - 23A - 40I = O$

DETERMINANTS

$(1M \times 1 + 2M \times 1 + 3M \times 1 + 4M \times 2 + 5M \times 1 = 19(6)$

ONE MARK QUESTIONS

1. Find the value of x if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$
2. Find the value of x if $\begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} x & 8 \\ 1 & x \end{vmatrix}$
3. Find the value of x if $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

4. If $A = \begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix}$ then show that $|2A| = 4|A|$
5. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then find $|3A|$
6. If A is a square matrix of order 3 and $|A| = 4$ then find $|2A|$.
7. If $|A\ | = 8$ then find $|AA'|$
8. If A is a 3×3 matrix and $|A| = 8$, find $|adj\ A|$
9. If A is a 3×3 matrix and $|A| = 3$, find $|adj\ A|$
10. If the matrix $\begin{pmatrix} 3 & x \\ 2 & 4 \end{pmatrix}$ is singular then find x.
11. If A is invertible matrix of order 2, then find $|A^{-1}|$.
12. If A be a square matrix of order 3×3, then find $|\ kA|$.

TWO MARKS QUESTIONS

1. Using determinants find the area of the triangle whose vertices are
   1) $(3,8), (-4,2), (5,1)$ 2) $(1,0), (6,0), (4,3)$
   3) $(2,7), (1,1), (10,8)$ 4) $(-2,-3), (3,2), (-1,-8)$
2. Using determinants find the equation of the line passing through the points
   1) $(1,2)$ and $(3,6)$ 2) $(3,1)$ *and* $(9,3)$
3. Find the value of K if area of the triangle is 4 sq. units and vertices are
   1) $(k, 0), (4,0), (0,2)$ 2) $(-2,0), (0,4), (0, k)$
4. If the area of the triangle with vertices $(2, -6)$and $(5,4)$ and (k,4) is 5 sq. units. Find the value of k using determinant method.
5. Find the equation of the line joining A(1, 3) and B (0, 0) using determinants. and find k if D(k, 0) is a point such that area of triangle ABD is 3sq units.
6. Show that the points $(a, b + c), (b, c + a)$ *and* $(c, a + b)$ are collinear.

THREE MARKS QUESTIONS

1. Examine the consistency and inconsistency of $x + 3y = 5$ *and* $2x + 6y = 8$.
2. Examine the consistency and inconsistency of $2x - y = 5$ *and* $x + y = 4$.
3. Examine the consistency and inconsistency of $x + 2y = 2$ *and* $2x + 3y = 3$.
4. If A be any given square matrix of order n, then A(*adj* A) = (*adj* A) A = A I , where I is the identity matrix of order n.
5. If A = $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$,then verify that A *adj* A = |A| I

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FOUR MARKS QUESTIONS

1. Prove the following

1. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

2. $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)\,(a+b+c)$

3. $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$

4. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

5. $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2\,(x+y+z)^3$

6. $\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix} = (x-p)(x-q)(x+p+q)$

7. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

8. $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

9. $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

10. $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$

11. $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

12. $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

2.1) Show that the matrix A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2-4A+I = O$, where I is 2 x 2 identity matrix and 2 x 2 zero matrix. Using this equation, find A^{-1}.

2) If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1}.

FIVE MARKS QUESTIONS

1. Solve the following system of linear equations using matrix method

   1) $3x - 2y + 3z = 8\,, 2x + y - z = 1$ and $4x - 3y + 2z = 4$

   2) $x - y + z = 4\,, 2x + y - 3z = 0$ and $x + y + z = 2$

   3) $2x + 3y + 3z = 5\,, x - 2y + z = 4$ and $3x - y - 2z = 3$

   4) $x - y + 2z = 7, 3x + 4y - 5z = -5$ and $2x - y + 3z = 12$

   5) $2x + y + z = 1\,, x - 2y - z = \frac{3}{2}$ and $3y - 5z = 9$

   6)If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations $2x - 3y + 5z = 11\,, 3x + 2y - 4z = -5$ and $x + y - 2z = -3$

CONTINUITY AND DIFFERENTIATION

$(1M \times 2 + 2M \times 3 + 3M \times 3 + 4M \times 1 + 5M \times 1 = 26(10)$

ONE MARK QUESTIONS

1. If $y = e^{\log x}$, then show that $\frac{dy}{dx} = 1$.
2. Differentiate the following with respect to x

   1) $y = \tan(2x + 3)$ 2) $y = \sin[cosx]$ 9)y= sin(logx)

   3) $y = \sin\,(x^2 + 5)$ 4) $y = \cos\sqrt{x}$ 10) y $=\cos^{-1}(e^x)$

   5) y =sin(ax+b). 6) y = $\sin(x^2)$ 11)y = e^{e^x}

   7) $y = e^{x^3}$. 8) y = xcosx

3. Find the derivative of $5cosx - 3sinx$ with respect to x.
4. Find the derivative of $e^{sin^{-1}x}$ with respect to x.
5. Find $\frac{dy}{dx}$if $x - y = \pi$.
6. Find $\frac{dy}{dx}$if $2x + 3y = \sin x$
7. Find $\frac{dy}{dx}$, if $y = \sin^{-1}(x\sqrt{x})$.

TWO MARK QUESTIONS

1. If $y = \log_{x^2}(\log x)$ then find $\frac{dy}{dx}$
2. If $y = \log_7 x\,[\log x]\,, find\, \frac{dy}{dx}$
3. If $y + \sin y = \cos x\,,$ $find$ $\frac{dy}{dx}$.
4. *If i)* $ax + by^2 = \cos y$ *ii)* $xy + y^2 = \tan x + y$ *iii)* $x^2 + xy + y^2 = 100$
   iv) $x^3 + x^2y + xy^2 + y^3 = 81$ v) $\sin^2 y + \cos xy = k$ vi) $\sin^2 x + \cos^2 y = 1, find\ \frac{dy}{dx}$
5. Examine the function f given by $f(x) = x^2$ is continuous at $x = 0$

6. If $\sqrt{x}+\sqrt{y}=\sqrt{10}$ then show that $\frac{dy}{dx}+\sqrt{\frac{y}{x}}=0.$

7. If $y=\sqrt{\frac{1-\cos 4x}{1+\cos 4x}}$ then prove that $\frac{dy}{dx}=2\sec^2(2x)$

8. If $y=\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ then prove that $\frac{dy}{dx}=\frac{1}{2}$

9. If $y=\tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right]$ then find $\frac{dy}{dx}$

10. If $y=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0<x<1$ then find $\frac{dy}{dx}$

11. Find $\frac{dy}{dx}$, if $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ $0<x<\frac{1}{\sqrt{2}}$.

12. If $y=\sin^{-1}(2x\sqrt{1-x^2})$ then prove that $\frac{dy}{dx}=\frac{2}{\sqrt{1-x^2}}$

13. If $y=\sin(2\sin^{-1}x)$ then prove that $\frac{dy}{dx}=2\sqrt{\frac{1-y^2}{1-x^2}}$

14. Find $\frac{dy}{dx}$, if $2x+3y=\sin y$.

15. Find $\frac{dy}{dx}$, if $ax+by^2=\cos y$.

16. Differentiate $\cos^{-1}(2x^2-1)$ $w.r.t.\cos^{-1}x$.

THREE MARKS QUESTIONS

1. If $y=f(x)$ is differentiable at $x=c$ then prove that it is continuous at $x=c$.
2. Prove that the function $f(x)=|x-1|$ is not differentiable at $x=1$
3. If $y=\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ then prove that $\frac{dy}{dx}=\frac{2^{x+1}\log 2}{1+4^x}$
4. If $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ prove that $\frac{dy}{dx}=\frac{1}{2(1+x^2)}$
5. Find $\frac{dy}{dx}$ if $\sin^2 y+\cos(xy)=k$
6. If $\cos y=x\cos(a+y)$ then prove that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$
7. If $x\sqrt{1+y}+y\sqrt{1+x}=0$ & $x\neq y$, then prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$
8. If $x=at^2, y=2at$ show that $\frac{dy}{dx}=\frac{1}{t}$
9. If $x=\sqrt{a^{\sin^{-1}t}}, y=\sqrt{a^{\cos^{-1}t}}$ then prove that $\frac{dy}{dx}=-\frac{y}{x}$
10. If $y=x^x$ *then find* $\frac{dy}{dx}$ ii) If $y=x^{sinx}$ *then find* $\frac{dy}{dx}$
11. $y=x^x-2^{sinx}$ then find $\frac{dy}{dx}$
12. If $y=(\sin x)^{\cos x}$ then find $\frac{dy}{dx}$
13. Differentiate $(\log x)^{\cos x}$ with r. to x.

14. If $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ then find $\frac{dy}{dx}$

15. i)If $x^y = y^x$ then find $\frac{dy}{dx}$. ii)If $(cosx)^y = (cosy)^x$ then find $\frac{dy}{dx}$

16. If $x = a\,(3\cos t - 4\cos^3 t)$, $y = a(3\sin t - 4\sin^3 t)$ then find $\frac{dy}{dx}$

17. If $x = \mathrm{a}\left(\cos \mathrm{t} + \log \tan \frac{\mathrm{t}}{2}\right)$, $\mathrm{y} = \mathrm{a}\sin \mathrm{t}$ prove that $\frac{\mathrm{dy}}{\mathrm{dx}} = \tan \mathrm{t}$.

18. Find $\frac{dy}{dx}$, if $x = a\,(\theta + \sin\theta)$, $y = a\,(1 - \cos\theta)$.

19. Differentiate $\sin^2 x$ with respect to $e^{\cos x}$.

20. If $y = \sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$ then find $\frac{dy}{dx}$

21. Verify Rolle's Theorem for the function $y = x^2 + 2, in\ [-2,,2]$

22. Verify Rolle's Theorem for the function $f(x) = x^2 + 2x - 8\,, x \in [-4,2]$

23. Verify Mean value theorem for the function $f(x) = x^2 in\ [2,4]$

24. Verify Mean value theorem for the function $f(x) = x^2 - 4x + 3\ in\ [1,4]$

25. Verify Mean value theorem for the function $f(x) = x^3 - 5x^2 - 3x$ in the interval$[a, b]$where $a = 1, b = 3$. Find all $c \in (1,3)$ for which $f'(c) = 0$

FOUR MARKS QUESTIONS

1. Find the value of K, so that the function

   1) $f(x) = \begin{cases} kx + 1 & if \quad x \le 5 \\ 3x - 5 & if \quad x > 5 \end{cases}$ is continuous at $x = 5$

   2) $f(x) = \begin{cases} kx + 1 & if \quad x \le \pi. \\ \cos x & if \quad x > \pi \end{cases}$ is continuous everywhere

   3) $f(x) = \begin{cases} \frac{k\, cosx}{\pi - 2x} & if \quad x \ne \frac{\pi}{2} \\ 3 & if \quad x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$

   4) $f(x) = \begin{cases} 2x + 1 & if\ x \le 3 \\ kx - 23 & if\ 3 < x < 5 \\ x^2 + 2 & if\ x \ge 5 \end{cases}$ is continuous everywhere.

   5) $f(x) = \begin{cases} \frac{1 - cos2x}{1 - \cos x}, & x \ne 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$.

   6) $f(x) = \begin{cases} Kx^2 & if\ x \le 2 \\ 3 & if\ x > 2 \end{cases}$ is continuous at $x = 2$.

2. For what value of λ is the function defined by $f(x) = \begin{cases} \lambda(x^2 - 2x) & if\ x \le 0 \\ 4x + 1 & if\ x > 0 \end{cases}$ continuous at $x = 0$? What about continuity $at\ x = 1$.

3. Find the value of $a \text{ and } b$ such that the function $f(x) = \begin{cases} 5 & if\ x \leq 2 \\ ax + b & if\ 2 < x < 10 \\ 21 & if\ x \geq 10 \end{cases}$

is continuous function.

FIVE MARKS QUESTIONS

1. If $y = A\sin x + B\cos x$ then prove that $\frac{d^2y}{dx^2} + y = 0$
2. If $y = (\tan^{-1} x)^2$ then prove that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$
3. If $\mathrm{y} = \sin^{-1}\mathrm{x}$ then prove that $(1 - \mathrm{x}^2)\mathrm{y}_2 - \mathrm{xy}_1 = 0$
4. If $y = 3\cos(\log x) + 4\sin(\log x)$ then prove that $x^2y_2 + xy_1 + y = 0$
5. If $e^y(x + 1) = 1$, prove that $\frac{dy}{dx} = -e^y$ and hence prove that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$
6. If $y = 500\,e^{7x} + 600\,e^{-7x}$ then prove that $y_2 = 49y$
7. If $y = Ae^{mx} + Be^{nx}$ then prove that $\frac{d^2y}{dx^2} - (m + n)\frac{dy}{dx} + mny = 0$
8. If $y = 3e^{2x} + 2e^{3x}$ then prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.

APPLICATION OF DIFFERENTIATION

$(\mathbf{2}\boldsymbol{M} \times \mathbf{1} + \mathbf{3}\boldsymbol{M} \times \mathbf{1} + \mathbf{5}\boldsymbol{M} \times \mathbf{1} + \mathbf{6}\boldsymbol{M} \times \mathbf{1} = \mathbf{16}(\mathbf{4})$

TWO MARK QUESTIONS

1. Find the slope of the tangent to the curve y=x^3−x at x=2.
2. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$.
3. Find the slope of the tangent to curve $y = x^3 - x + 1$ at the point whose $x -$ coordinate is 2.
4. Prove that the function f given by $f(x) = x^2e^{-x}$is increasing in(0,2).
5. Using differentials, find the approximate value of each of the following

   1) $\sqrt{36.6}$ 2) $(25)^{\frac{1}{3}}$ 3) $(0.009)^{\frac{1}{3}}$
6. Find the approximate change in the volume V of a cube of side x m caused by increasing the side by 1%.
7. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its volume.
8. Find the approximate change in the surface area of a cube of side x meters caused by decreasing the side by 1%.
9. Find the slope of the tangent and normal to the curve $x = 3t^2 + 1, y = t^3 - 1\ at\ t = 1$.
10. Find the slope of tangent to the curve $y = x^3 - 3x + 2$ at the point whose$x\ coordinate$is 3.

11. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$.

12. Find the point on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are parallel to $x - axis$.

13. Find points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are parallel to $y -$ axis.

14. Find the interval in which the function f given by $f(x) = 2x^2 - 3x$ is strictly increasing.

15. Find the local maximum value of the function $g(x) = x^3 - 3x$.

16. Find two positive numbers x and y such that their sum is 35 and the product x^2y^5 is a maximum.

17. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

THREE MARKS QUESTIONS:

1. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the $x -$ axis.
2. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$.
3. Find the equations of the tangent and normal to the given curve $y = x^3$ at $(1,1)$.
4. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the $y -$coordinate of the point.
5. Find the point at which the tangent to the curve $y = \sqrt{4x - 3}$ has its slope $\frac{2}{3}$
6. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1,2).
7. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.
8. Find the intervals in which the following functions are strictly increasing or strictly decreasing.

   1) $f(x) = x^2 + 2x - 5$ 2) $f(x) = 2x^2 - 3x$

   3) $f(x) = x^2 - 4x + 6$ 4) $f(x) = 2x^3 - 3x^2 - 36x + 7$
9. Show that the function $f(x) = \frac{\log x}{x}$ has minimum at $x = e$.
10. Find two numbers whose sum is 24 and whose product is as large as possible.

11. Find two positive numbers whose sum is 15 and the sum of whose square is minimum.

12. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

FIVE MARKS QUESTIONS:

1. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm
2. The length x of a rectangle is decreasing at the rate of 3 cm/min and the width y is increasing at the rate of 2 cm/min. When x= 10 cm and y= 6 cm, find the rate of change of (i) the perimeter and (ii) the area of the rectangle
3. The length x of a rectangle is decreasing at the rate of 5cm/min and the width y is increasing at the rate of 4cm/min. When x= 8 cm and y= 6 cm, find the rates of change of (i) the perimeter, (ii) the area of the rectang
4. The volume of a cube is increasing at the rate of 8cm3/s. How fast is the surface area increasing when the length of an edge is 12cm?
5. An edge of a variable cube is increasing at the rate of 3cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?
6. Sand is pouring from a pipe at the rate of 12cm3/s. The falling sand form a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?
7. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of 2 m/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?
8. A man of height 2 meters walks at a uniform speed of 5 km/hour, away from a lamp post which is 6 meters high. Find the rate at which the length of his shadow increases.
9. A particle moves along the curve $6y=x3+2$. Find the points on the curve at which the $y-$coordinate is changing 8 times as fast as the $x-$coordinate.

SIX MARKS QUESTIONS

1. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.
2. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

3. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.
4. The sum of the perimeter of a circle and square is k, where k is some constant.Prove that the sum of their areas is least when the side of square is double the radius of the circle.
5.Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.
6.Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

INTEGRATION

$(1M \times 2 + 2M \times 3 + 3M \times 3 + 5M \times 1 + 6M \times 1 = 28(10)$

ONE MARK QUESTIONS

1. Evaluate the following:

1) $\int x\sqrt{x}\ dx$
2) $\int (\sec x[\sec x + \tan x])\,dx$
3) $\int (\tan^2 x)\,dx$
4) $\int \left(\frac{1}{\sqrt{4-x^2}}\right) dx$
5) $\int (1-x)\sqrt{x}dx$
6) $\int \left(cosec\ x(cosec\ x + \cot x)\right) dx$
7) $\int \frac{x^3+5x^2-4}{x^2}dx$
8) $\int 4^{2x+1}\,dx$
9) $\int \cos 3x\,dx$
10) $\int e^x(1+\tan x+\tan^2 x)dx$
11) $\int e^x\left(\log x + \frac{1}{x}\right) dx$
12) $\int \sqrt{1+\cos 2x}dx$
13) $\int \mathrm{e}^{\mathrm{x}}\left(\frac{1}{\mathrm{x}}-\frac{1}{\mathrm{x}^2}\right)\,\mathrm{dx}$
14) $\int (2x-3\cos x+e^x).\,dx$
15) $\int\ (\sin x\ + \cos x)dx$
16) $\int (e^{2x+3})dx.$

18. $\int \tan^2 2x\ \mathrm{dx}$

19. $\int \sin(2+5x)\ \mathrm{dx}$

20.Find the anti-derivative of $\frac{1}{x\sqrt{x^2-1}}, x>1$ with respect to x

21.Find the anti derivative of $x^2\left(1-\frac{1}{x^2}\right)$ with respect to x.

22. Find the anti derivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ with respect to x.

TWO MARKS QUESTIONS

1. Find the anti-derivative F of the function $f(x) = 4x^3 - 6$, where $F(0) = 3$
2. Evaluate the following:

1) $\int \left(\frac{x^3-x^2+x-1}{x-1}\right) dx$

2) $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$

3) $\int \left(\frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}\right) dx$

4) $\int \frac{\sin^2 x}{1+\cos x} dx$

5) $\int \left(\frac{1}{1-\cos x}\right) dx$

6) $\int (\sin x + \cos x)^2 \, dx$

7) $\int \sin^2 x \, dx$

8) $\int \frac{1}{\sin^2 x \cos^2 x} dx$

9) $\int \frac{\sin(\tan^{-1} x)}{1+x^2} dx$

10) $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$

11) $\int \frac{dx}{e^x - 1}$

12) $\int \frac{2 - 3\sin x}{\cos^2 x} dx$

13) $\int \frac{(\log x)^2}{x} dx$

14) $\int_1^e \log x \, dx$

15) $\int x e^x dx$

16) $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx.$

17) $\int \frac{e^{2x} - 1}{e^{2x} + 1} dx$

18) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

19) $\int_0^\pi \left(\sin^2\left(\frac{x}{2}\right) - \cos^2\left(\frac{x}{2}\right)\right) dx$

20) $\int \frac{x^2}{1-x^6} \, dx$

21) $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} \, dx$

22) $\int_0^1 \frac{1}{1+x^2} . dx.$

23) $\int \frac{\sin^2 x}{1+\cos x} . dx$

24) $\int_1^e \frac{1}{x} . dx$

25) $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$

26) $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$

27) $\int \frac{x-1}{\sqrt{x^2-1}} dx$

27) $\int x^2 e^x dx$

29) $\int x \sec^2 x dx$

30) $\int_0^5 (x+1) dx$

THREE MARKS QUESTIONS

Evaluate the following:

1) $\int (\sin 2x \sin 3x) dx$

2) $\int \left(\frac{\sin x}{\sin(x-a)}\right) dx$

3) $\int \left(\frac{1}{\cos(x+a)\cos(x+b)}\right) dx$

4) $\int \frac{(x+1)(x+\log x)^2}{x} dx$

5) $\int \frac{\sin^3 x+\cos^3 x}{\sin^2 x \cos^2 x} dx$

6) $\int \frac{\cos x-\sin x}{1+\sin 2x} dx$

7) $\int \frac{1}{x+x\log x} dx$

8) $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$

9) $\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8} dx$

10) $\int \frac{\tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}} dx$

11) $\int \frac{x}{(x-1)(x-2)} dx$

12) $\int \frac{x}{(x-1)(x-2)(x-3)} dx$

13) Find $\int \frac{x^2}{(x^2+1)(x^2+4)} dx$

14) $\int \frac{1}{(x^2+1)(x^2+4)} dx$

15) $\int \frac{1}{x(x^n+1)} dx$

16) $\int \frac{x}{(x-1)^2(x+2)} dx$

17) $\int e^x \left(\frac{x}{(1+x)^2}\right) dx$

18) $\int \left(\log(\log x) + \frac{1}{(\log x)^2}\right) dx$

19) $\int \sin^{-1} x \, dx$

20) $\int x(\log x)^2 \, dx$

21) $\int x sin^{-1} x \, dx$

22) $\int x tan^{-1} x \, dx$

23) $\int e^x \sin x \, dx$

24) $\int \frac{(x^4-x)^{1/4}}{x^5} dx$

25) $\int e^x \left(\frac{2+\sin 2x}{1+\cos 2x}\right) dx$

26) $\int \frac{(x+1)(x+\log x)^2}{x} dx$

27) $\int \frac{x sin^{-1} x}{\sqrt{1-x^2}} dx$

1. Evaluate **the following as limit of sum: (Definite integrals)**

1) $\int_0^5 (x+1)dx$

2) $\int_2^3 (x^2)dx$

3) $\int_0^2 (x^2+1)dx$

4) $\int_0^2 (e^x)dx$

FIVE MARKS QUESTIONS

2. Provethe following (3MARK)

1) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

2) $\int \frac{dx}{\sqrt{a^2+x^2}} = \log\left|x + \sqrt{a^2+x^2}\right| + c$

3) $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x + \sqrt{x^2-a^2}\right| + c$

4) $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

5) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

6) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} log \left|\frac{x-a}{x+a}\right| + c$

7) $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} sin^{-1}\left(\frac{x}{a}\right) + c$

8) $\int \sqrt{a^2+x^2}dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log\left|x+\sqrt{a^2+x^2}\right| + c$

9) $\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c$

3. Evaluate the following: (2MARK)

1) $\int \frac{dx}{x^2-6x+13}$

2) $\int \frac{dx}{9x^2+6x+5}$

3) $\int \frac{dx}{3x^2+13x-10}$

4) $\int \frac{dx}{\sqrt{5x^2-2x}}$

5) $\int \frac{dx}{\sqrt{(x-1)(x-2)}}$

6) $\int \frac{dx}{\sqrt{x^2+2x+2}}$

7) $\int \frac{dx}{\sqrt{2x-x^2}}$

8) $\int \sqrt{4-x^2}\, dx$

9) $\int \sqrt{x^2+4x+6}\, dx$

10) $\int \sqrt{1-4x-x^2}\, dx$

11) $\int \sqrt{x^2+4x+1}\, dx$

12) $\int \sqrt{14x-20-2x^2}\, dx$

SIX MARKS QUESTIONS

1. Prove that $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ where $a < c < b$ and hence integrate the following.

1) $\int_{-5}^{5} |x+2|\, dx$

2) $\int_{-1}^{2} |x^3-x|\, dx$

2. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$and hence integrate the following.

(i) $\int_0^{\pi/4} \log(1+tanx)\, dx$

(ii) $\int_0^{\frac{\pi}{2}} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}} dx$

(iii) $\int_0^{\frac{\pi}{2}} (2logsinx - logsin2x)dx$

iv) $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$

(v) $\int_0^{\frac{\pi}{2}} \frac{cos^5x}{sin^5x+cos^5x} dx$

(vi) $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx$

(vii) $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$

(viii) $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$

3. Prove that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$& hence integrate $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}} dx$

4. Prove that $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & if \quad f(2a-x) = f(x) \\ 0 & if \quad f(2a-x) = -f(x) \end{cases}$

and hence integrate $\int_0^{2\pi} \cos^5 x\, dx$

5.Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & if \quad f(x) \text{ } is \text{ } even \\ 0 & if \quad f(x) \text{ } is \text{ } odd \end{cases}$ and

integrate the

1) $\int_{-1}^{1} \sin^5 x \cos^4 x \, dx$

2) $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx$

3) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + xcosx + tan^5 x)dx$

APPLICATION OF INTEGRATION

$(3M \times 1 + 5M \times 1 = 8(2)$

THREE MARKS QUESTIONS:

1. Find the area of the region bounded by the curve $y^2 = x$ and the lines x=1, x=4 and the x-axis in the first quadrant.
2. Find the area of the region bounded by the curve $y^2 = x$ and the line $x = 1, x = 4$ and the x-axis in the first quadrant.
3. Find the area of the region bounded by $y^2 = 9x, x = 2, x = 4$ and the x-axis in the first quadrant.
4. Find the area of the region bounded by $x^2 = 4y, y = 2, y = 4$ and the y-axis in the first quadrant.
5. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.
6. Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.
7. Find the area of the parabola $y^2 = 4ax$ bounded by its latus rectum.
8. Find the area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$

FIVE MARKS QUESTIONS

1. Find the area enclosed by the circle $x^2 + y^2 = a^2$
2. Find the area enclosed by the circle $x^2 + y^2 = 9$
3. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
4. Find the area enclosed by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

5. Find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the ordinates x=0 and x=ae,
where $b^2=a^2(1-e^2)$ and e<1.
6. Find the area of the region bounded by the parabola $x^2 = 4y$ and the line $x = 4y - 2$.
7. Find the area of the region in the first quadrant enclosed by the $x-axis$, the line $y = x$ and the circle $x^2 + y^2 = 32$.
8. Find the area of the smallest part of the circle $x^2 + y^2 = a^2$ cutoff by the line $x = \frac{a}{\sqrt{2}}$
9. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$
10. Find the area of the region bounded by the parabolas $y = x^2$and $y^2 = x$.
11. Using integration find the area of the region bounded by the triangle whose vertices are $(1,0), (2,2) \& (3,1)$.
12. Using integration find the area of the region bounded by the triangle whose vertices are $(-1,0), (1,3) \& (3,2)$.
13. Using integration find the area of the region bounded by the triangle whose sides have the equation $y = 2x + 1, y = 3x + 1, x = 4$.
14. Prove that the curve $y^2 = 4x \; \& x^2 = 4y$ divides the area of the square bounded by $x = 0, x = 4, y = 4$ and $y = 0$ into three equal parts.
15. Find the area bounded between the circles $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$

DIFFERENTIAL EQUATIONS

$(2M \times 1 + 3M \times 2 + 5M \times 1 = 13(4)$

TWO MARKS QUESTIONS:

1. **Find the order and degree of the following differential equations:**

1) $\frac{dy}{dx} - \cos x = 0$

2) $xy\left(\frac{d^2y}{dx^2}\right) + x\left(\frac{dy}{dx}\right)^2 - y\left(\frac{dy}{dx}\right) = 0$

3) $\frac{d^4y}{dx^4} + \sin\left(\frac{d^3y}{dx^3}\right) = 0$

4) $\left(\frac{d^2y}{dx^2}\right) + \cos\left(\frac{dy}{dx}\right) = 0$

5) $y''' + y^2 + e^{y'} = 0$

6) $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$

7) $\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$ 8) $(y^{111})^2 + (y^{11})^3 + (y^1)^4 + y^5 = 0$

THREE MARKS QUESTIONS:

1. Form the differential equation representing the family of curves y = a sin (x + b), where a, b are arbitrary constants.
2. Form the differential equation of the family of curves $\frac{x}{a} + \frac{y}{b} = 1$, by eliminating the constants "a" and "b"
3. Form the differential equation representing the family of straight line passes through the origin.
4. Form the differential equation of family of curve $y = ae^{3x} + be^{-2x}$
5. Form the differential equation representing the given family of curves $y=e^{3x}(a + bx)$ by eliminating the arbitrary constants.
6. Form the differential equation representing the family of ellipses having foci on x-axis and centre at the origin.
7. Form the differential equation of the family of circles touching the x-axis at origin.
8. Form the differential equation of the family of circles touching the y-axis at origin.
9. Form the differential equation of the family of circles having centre on $y - axis$ and radius 3 units.
10. Find the general solution of the following differential equations

    1) $\frac{dy}{dx} = \frac{x+1}{2-y}$ 2) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

    3) $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$ 4) ylogy dx – x dy = 0.

    5) $\sec^2 x \tan y \; dx + \sec^2 y \tan x \, dy = 0$ 6) $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$

    7) $(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$ 8) $\frac{dy}{dx} + y = 1 \;\; (y \neq 1)$

    9) $e^x \tan y dx + (1 - e^x) \sec^2 y \, dy = 0$
11. Find the equation of the curve passing through the point (1,1) whose differential equation is $xdy = (2x^2 + 1)dx$
12. Find the equation of the curve passing through the point (0,0) whose differential equation is $\frac{dy}{dx} = e^x \sin x$
13. Find the equation of a curve passing through the point (-2,3) given that the slope of the tangent to the curve at any point (x ,y) is $\frac{2x}{y^2}$.

FIVE MARKS QUESTIONS:

1. Find the general solution of the following differential equations

1) $\frac{dy}{dx} - y = \cos x$

2) $x\frac{dy}{dx} + 2y = x^2$

3) $ydx - (x + 2y^2)dy = 0$

4) $\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x$

5) $\frac{dy}{dx} + 2y = \sin x$

6) $\frac{dy}{dx} + (\sec x)y = \tan x$

7) $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$

8) $\cos^2 x\frac{dy}{dx} + y = \tan x$

9) $x\frac{dy}{dx} + 2y = x^2 \log x$

10) $(1 + x^2)dy + 2xydx = \cot x\, dx$

11) $(x + y)\frac{dy}{dx} = 1$

12) $(x + 3y^2)\frac{dy}{dx} = y$

13) $(\tan^{-1} y - x)dy = (1 + y^2)dx$

VECTORS

$(\mathbf{1}M \times 2 + \mathbf{2}M \times 2 + \mathbf{3}M \times 2 = \mathbf{12}(6)$

ONE MARK QUESTIONS

1. Definitions
   i)Negative vectors ii)Collinear Vectors iii)Coplanar vectors .
2. Find the magnitude of the vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$
3. Write two different vectors having same magnitude.
4. Find the unit vector in the direction of $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$
5. Find the unit vector in the direction of $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$
6. Find the angle between two vectors $\vec{a}\&\vec{b}$ with magnitude 1 and 2 respectively and there dot product is 1.
7. If $|\vec{a}| = 3$, $|\vec{b}| = \frac{\sqrt{2}}{3}\&$ $|\vec{a} \times \vec{b}| = 1$ then find the angle between $\vec{a}$ and $\vec{b}$.
8. Find the value of m if the following vectors are orthogonal
   i)$3\hat{\imath} + 2\hat{\jmath} + 8\hat{k}\&2\hat{\imath} + m\hat{\jmath} + \hat{k}$ ii) $3\hat{\imath} + \hat{\jmath} - \hat{k}\&\hat{\imath} + m\hat{\jmath} - 3\hat{k}$

TWO MARKS QUESTIONS

1. Find the position vector of the point which divides the join of the points with position vectors are $2\vec{a} - 3\vec{b}\&3\vec{a} - 2\vec{b}$ internally & externally in the ratio 2:3.
2. Find the position vector of the point P which divides the line AB internally in the ratio $3:2$ where position vectors of A and B are $2\vec{a} + 3\vec{b}\&3\vec{a} - \vec{b}$.
3. Prove that $(3\vec{a} - 2\vec{b}) \times (2\vec{a} + \vec{b}) = 7(\vec{a} \times \vec{b})$
4. Find the cosine angle between the following vectors :
   i)$\vec{a} = \hat{\imath} + \hat{\jmath} - \hat{k}\&\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ ii)$\vec{a} = -\hat{\imath} - 2\hat{\jmath} + \hat{k}\&\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$
5. Find the angle between the vectors$\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$and $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$.

6. Show that the following vectors are perpendicular or orthogonal
   i) $2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$&$4\hat{\imath} - 2\hat{\jmath} - \hat{k}$ ii) $2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$&$3\hat{\imath} + 6\hat{\jmath} + 6\hat{k}$
9. Find the projection of $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ on $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$
10. Find the projection of $\vec{b}$ on $\vec{a}$ where $\vec{a} = \hat{\imath} - \hat{\jmath}$ and $\vec{b} = \hat{\imath} + \hat{\jmath}$
11. If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$, find $|\vec{a} - \vec{b}|$.
12. If $\vec{a} = 5\hat{\imath} - \hat{\jmath} - 3\hat{k}$ and $\vec{b} = \hat{\imath} + 3\hat{\jmath} - 5\hat{k}$, then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.
13. Find the area of the parallelogram whose adjacent sides are
   i) $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$&$\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$ ii) $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$&$\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$
14. If $\vec{a}$ is a unit vector and $(\vec{x} + \vec{a}).(\vec{x} - \vec{a}) = 12$ then find $|\vec{x}|$
15. Find $|\vec{b}|$, if $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$.
16. Find the volume of the Parallelopiped if the co-initial edges are
   i) $2\hat{\imath} + \hat{\jmath} - \hat{k},\ 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}\ and\ \hat{\imath} - 3\hat{\jmath} - 3\hat{k}$

THREE MARKS QUESTIONS

1. Find the position vector of a point which divides the line AB in the ratio m:n internally.
2. The position vectors of two points P & Q are $\hat{\imath} + 2\hat{\jmath} - \hat{k}$&$-\hat{\imath} + \hat{\jmath} + \hat{k}$ respectively. Find the position vector of a point R which divides the line PQ in the ratio 2:1
   i) internally ii) externally
3. If $\vec{a}, \vec{b}, \vec{c}$ are unit vector such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.
4. If three vectors $\vec{a}, \vec{b}, \vec{c}$ are such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ if $|\vec{a}| = 1$, $|\vec{b}| = 4$, $|\vec{c}| = 2$
5. If $\vec{a}$,$\vec{b}$&$\vec{c}$ are three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$& each vector is orthogonal to sum of the other two vectors then find $|\vec{a} + \vec{b} + \vec{c}|$.
6. If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$&$\vec{c} = 3\hat{\imath} + \hat{\jmath}$ such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$ then find λ.

7. The two adjacent sides of a parallelogram are $2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$, $\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$ then find the unit vector parallel to its diagonal and also find its area.
8. The scalar product of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$&$\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to one. Find the value of λ.
9. Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$&$\vec{a} - \vec{b}$ where $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$.

10. Find the area of the triangle with vertices are (1,-1,2), (2,1,-1) & (3,-1,2).
11. Show that the points A(2,3,-1), B(1,-2,3), C(3,4,-2) and D(1,-6,6) are coplanar.
12. Show that the four points with position vectors $4\hat{\imath} + 8\hat{\jmath} + 12\,\hat{k}$, $2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$, $3\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$ and $5\hat{\imath} + 8\hat{\jmath} + 5\hat{k}$ are coplanar.
13. Find λ if the vectors $\vec{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$ and $\vec{c} = \lambda\hat{\imath} + 7\hat{\jmath} + 3\hat{k}$ are coplanar.
14. For any three vectors $\vec{a}$, $\vec{b}$, &$\vec{c}$ Prove that $\left[\vec{a} + \vec{b}\vec{b} + \vec{c}\vec{c} + \vec{a}\right] = 2\left[\vec{a}\vec{b}\vec{c}\right]$.
15. Show that the vectors $\vec{a}$, $\vec{b}$, &$\vec{c}$ are coplanar if $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$, &$\vec{c} + \vec{a}$ are coplanar.

THREE DIMENSIONAL GEOMETRY

$(\mathbf{1}M \times 1 + \mathbf{2}M \times 2 + \mathbf{3}M \times 1 + \mathbf{5}M \times 1 = \mathbf{13}(5))$

ONE MARKS QUESTIONS:

1. Find the direction cosines of $x - axis$.
2. Find the direction cosines of $y - axis$.
3. Find the direction cosines of $z{-}axis$.
4. If a line has direction ratios $-18,12,-4$ determine its direction cosines.
5. If a line makes angle $90^0, 60^0$ *and* 30^0 with positive direction of x, y *and* z axis respectively. Find its direction cosines.
6. Find the direction cosines of a line which makes equal angles with co-ordinate axes.
7. Find the direction ratio of the line $\frac{x-1}{2} = 3y = \frac{2z+3}{4}$
8. Find the Cartesian equation of the plane $\vec{r}.(\hat{\imath} + \hat{\jmath} - \hat{k}) = 2$

9. Find the equation of the plane with intercepts 1,2,-3 on the co-ordinate axes.
10. Find the equation of the plane with the intercept 2,3 and 4 on x, y and z axes respectively.
11. Find the equation of the plane with intercepts 3 on the y axis and parallel to ZOX plane.
12. Find the intercepts cut off by the place $2x + y - z = 5$.
13. Find the distance of a point.(3,-2,1) to the plane $2x - y + 2z + 3 = 0$.
14. Find the distance of a point (2,5,-3) from the plane $\vec{r}.(6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) = 4$.
15. Find the distance of the plane $2x - 3y + 4z - 6 = 0$ from the origin.

TWO MARKS QUESTIONS

1. Find the vector equations of the line through the point (5, 2, – 4) and which is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$.
2. Find the Cartesian equations of the line through the point (5, 2, – 4) and which is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$.
3. Find the vector equation for the line passing through the points i) (–1, 0, 2) and (3, 4, 6). ii) (3,-2,-5) & (3,-2,6).
4. Find the i)vector, ii) Cartesian equations of the line through the point (1,2,3) and which is parallel to the vector $3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$.
5. Find the i) vector ii) Cartesian equations of the line which passes through the point (-2,4,-5) and parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.
6. Find the equation of a line in i)vector ii)Cartesian forms which passing through (2,3,-4) & having the direction ratios are 1,2,3.
7. Find the angle between the pair of lines given by $\hat{r} = 3\hat{i} + 2\hat{j} - 2\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\hat{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$
8. Find the angle between the pair of lines $\frac{x+3}{3} = \frac{y-1}{3} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.
9. Find the vector and Cartesian equations of the plane which passes through the point (5,2,-4) and perpendicular to the line with direction ratios 2,3,-1.
10. Find the vector equations of the plane passing through the points R(2, 5, – 3), S(– 2, – 3, 5) and T(5, 3,– 3).
11. Find the angle between the two planes i) $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$ ii) $3x - 6y + 2z = 7$ and $2x + 2y - 2z = 5$.
12. Find the i)vector ii) Cartesian equations of the plane passes through the point i)(1,4,6) and the normal to the plane is $\hat{\imath} - 2\hat{\jmath} + \hat{k}$.
    ii)(1, 0, – 2) and the normal to the plane is $\hat{\imath} + \hat{\jmath} - \hat{k}$
13. Find the angle between the planes whose vector equations are

$\vec{r}.\left(2\hat{\imath}+2\hat{\jmath}-3\hat{k}\right)=5$and $\vec{r}.\left(3\hat{\imath}-3\hat{\jmath}+5\hat{k}\right)=3$.

THREE MARKS QUESTIONS:

1. Find the shortest distance between the lines
   i) $\hat{r}=\hat{\imath}+2\hat{\jmath}+\hat{k}+\lambda\left(\hat{\imath}-\hat{\jmath}+\hat{k}\right)$and $\hat{r}=2\hat{\imath}-\hat{\jmath}-\hat{k}+\mu(2\hat{\imath}+\hat{\jmath}+2\hat{k})$.
   ii) $\hat{r}=\hat{\imath}+\hat{\jmath}+\lambda\left(2\hat{\imath}-\hat{\jmath}+\hat{k}\right)$ and $\hat{r}=2\hat{\imath}+\hat{\jmath}-\hat{k}+\mu(3\hat{\imath}-5\hat{\jmath}+2\hat{k})$.
   iii) $\hat{r}=\hat{\imath}+2\hat{\jmath}+3\hat{k}+\lambda(\hat{\imath}-3\hat{\jmath}+2\hat{k})$and $\hat{r}=4\hat{\imath}+5\hat{\jmath}+6\hat{k}+\mu(2\hat{\imath}+3\hat{\jmath}+\hat{k})$.
2. Find the shortest distance between the lines
   $\frac{x+1}{7}=\frac{y+1}{-5}=\frac{z+1}{1}$ and $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$.
3. Find the distance between the lines whose vector equations are
   $\hat{r}=\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$ and $\hat{r}=3\hat{\imath}+3\hat{\jmath}-5\hat{k}+\mu(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$.
4. Find the vector equation of the plane passing through the intersection of the planes $\hat{r}=(\hat{i}+\hat{j}+\hat{k})=6$ and $\hat{r}=(2\hat{i}+3\hat{j}+4\hat{k})=-5$, and the point (1, 1, 1).
5. Find the equation of the plane through the line of intersection of the planes $3x-y+2z-4=0$ & $x+y+z-2=0$ & passing through the point (2,2,1)
6. Find the equation of the plane through the line of intersection of the planes $\vec{r}.\left(2\hat{\imath}+2\hat{\jmath}-3\hat{k}\right)=7$ &$\vec{r}.\left(2\hat{\imath}+5\hat{\jmath}+3\hat{k}\right)=9$ a& passing through the point (2,1,3)
7. Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$& the plane$10x+2y-11z=3$.

FIVE MARKS QUESTIONS:

1. Derive the equation of the line in space, passing through a point and parallel to a vector both in vector and Cartesian forms.
2. Derive the equation of the line in space, passing through two points both in vector and Cartesian forms.
3. Derive the equation of a plane in Normal form in both vector and Cartesian form.
4. Derive the equation of a plane passing through a point and perpendicular to a vector in both vector and Cartesian form.
5. Derive the equation of a plane passing through three non collinear points in both vector and Cartesian form.

LINEAR PROGRAMMING

$(\mathbf{1M}\times\mathbf{1}+\mathbf{6M}\times\mathbf{1}=\mathbf{7(2)})$

ONE MARK QUESTIONS

Definitions of terms used in LPP:

1. Objective function:
2. Constraints
3. Optimisation problem:
4. Feasible region:
5. Feasible solutions:
6. Optimal solution:

SIX MARKS QUESTIONS

1. Minimise Z = – 3x + 4y subject to x + 2y ≤ 8, 3x + 2y ≤ 12, x ≥ 0, y ≥ 0
2. Maximise Z = 5x + 3y subject to 3x + 5y ≤ 15, 5x + 2y ≤ 10, x ≥ 0, y ≥0.
3. Maximise Z = 3x + 2y subject to x + 2y ≤ 10, 3x + y ≤ 15, x, y ≥ 0.
4. Minimise and Maximise Z = 5x + 10y subject to
   x + 2y ≤ 120, x + y ≥60, x – 2y ≥ 0, x, y ≥ 0.
5. Minimise and Maximise Z = x + 2y subject
   to x + 2y ≥ 100, 2x – y ≤ 0, 2x + y ≤ 200; x, y ≥ 0.

PROBABILITY

$(1M \times 1 + 2M \times 1 + 3M \times 1 + 5M \times 2 = 12(5)$

ONE MARKS QUESTIONS:

1. If A is an event of a sample space 'S" of an random experiment then prove that $P(S|A) = 1$ Where $P(A) \neq 0$
2. If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ *and* $P(A \cap B) = \frac{4}{13}$ then find $P(A|B)$
3. If $P(E) = 0.6$ and $P(E \cap F) = 0.2$ then find $P(F/E)$
4. If P(A)= ½ , P(B)=0 then find P(A/B).
5. The random variable X has probability distribution $P(X)$ of the following form.

$$P(X) = \begin{cases} k & if \quad x = 0 \\ 2k & if \quad x = 1 \\ 3k & if \quad x = 2 \\ 0 & otherwise \end{cases}$$

6. If $P(A) = \frac{8}{15}, P(B) = \frac{2}{15}$ *and* $P(A \cap B) = \frac{7}{15}$ then find $P(B|A)$
7. If $P(B) = 0.5$ *and* $P(A \cap B) = 0.32$ then find $P(A|B)$
8. If $P(A) = \frac{3}{5}, P(B) = \frac{1}{5}$ then find $P(A \cap B)$ if A & B are independent events.
9. If A and B are independent events with $P(A) = 0.3$ and $P(B) = 0.4$, find $P(A \cap B)$.
10. An urn contains 5 red and 2 black balls. Two balls are randomly selected. Let X represents the number of black balls, what are the possible values of X?

TWO MARKS QUESTIONS:

1. Two coins are tossed once, find $P(E/F)$ where E: no tail appears, F : no head appears.

2. If A and B are independent events with $P(A) = 0.3, P(B) = 0.4$ then find $P(A \cup B)$
3. A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F is the event 'the number appearing is even', then find whether E and F are independent
4. Two cards drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.
5. Given that the event A and B are such that $P(A) = \frac{1}{2}$, $P(A \cap B) = \frac{3}{5}$ & $P(B) = k$, find k if A and B are independent events.
6. If the probability distribution of X is

X	0	1	2	3	4
P(X)	0.1	K	2k	2k	k

then find the value of k.

7. The random variable X has a probability distribution P(X) of the followingform where k is some number: $P(x) = \begin{cases} k & if\ x = 0 \\ 2k & if\ x = 1 \\ 3k & if\ x = 2 \\ 0 & otherwise \end{cases}$, determine the value of k and $P(x \leq 2)$
8. Find the probability distribution of number of heads in two tosses of a coin.

9.A random variable X has the following probability distribution

X	0	1	2	3	4	5	6	7
P(X)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

Determine i) K ii) P(X<3)

THREE MARKS QUESTIONS:

1. An urn contains 5 red & 5 black balls. A ball is drawn at random its color is noted and is returned to the urn. Moreover, 2 additional balls of the color drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?
2. A bag contains 4 red & 4 black balls, another bag contains 2 red & 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that it was drawn from first bag.

3. Bag I contains 3 red & 4 black balls and bag II contains 4 red & 5 black balls. One ball is transferred from bag I to bag II and then a ball is drawn from bag II. The drawn ball is found to be red. Find the probability that the transferred ball is black.
4. Given three identical boxes, I,II and III, each containing two coins. In box I, both coins are gold coins, In box II, both are silver coins and in box III, there is one gold coin and one silver coin. A person chooses a box at random & takes out a coin. If the coin is gold, what is the probability that the other coin in the box is also of gold?
5. An insurance company insured 2000 scooter drivers, 4000 car drivers & 6000 truck drivers. The probability of an accidents are 0.01, 0.03 & 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?
6. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.
7. Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. Find the probability that it is actually head.
8. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses the answer will be correct with probability $\frac{1}{4}$, what is the probability that the student knows the answer given that he answered it correctly?
9. Find the probability distribution of
   i) Number of heads in two tosses of a coin.
   ii) Number of tails in the simultaneous tosses of three coins.
   iii) Number of heads in four tosses of a coin.
10. Find the mean number of heads in three tosses of a coin.

FIVE MARKS QUESTION:

1. If a coin is tossed 8 times. Find the probability of
   i) At least five heads ii)At most five heads

2. A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of i) 5 successes ii) at most 5 successes iii)atleast 5 seccesses
3. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that i) All the 5 cards are spades? ii) Only 3 cards are spades? iii) None is a spade?
4. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize
   i) At least once ii) Exactly once iii) At least twice
5. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs
   i) None ii) Mora than one iii) Not more than one iv) At least one will fuse after 150 days of use.
6. A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F be the event 'the number appearing is even' then find whether E and F are independent ?
7. An unbiased die is thrown twice. Let the event A be 'odd number on the first throw' and B the event 'odd number on the second throw'. Check the independence of the events A and B.
8. Three coins are tossed simultaneously. Consider the event E 'three heads or three tails', F 'at least two heads' and G 'at most two heads'. Of the pairs (E,F), (E,G) and (F,G), which are independent? which are dependent?
9. Let A and B be independent events with P(A) = 0.3 and P(B) = 0.4. Find (i) $P(A \cup B)$ (ii) $P(A \cap B)$ (iii) $P(A|B)$ (iv) $P(B|A)$.
10. Given two independent events A and B such that P(A) = 0.3, P(B) = 0.6. Find (i) P(A and B) (ii) P(A and not B) (iii) P(A or B) (iv) P(neither A nor B).
11. Probability of solving specific problem independently by A and B are ½ and 1/3 respectively. If both try to solve the problem independently, find the probability that
   i)the problem is solved (ii) exactly one of them solves the problem.

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