Pairwise DαO - Connectedness in bitopological spaces

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***Abstract* — O.R.Sayed, A.M.Khalil [23] introduced the notion of Dα - closed sets in topological spaces in 2015.The aim of this paper is to introduce the notion of pairwise DαO - connected spaces and pairwise DαO - disconnected spaces.**

***Keywords* — Dα - closed , Dα - open, pairwise DαO - connected spaces and pairwise DαO - disconnected spaces.**

I. Introduction

Pervin [15] was first to define connectedness and components in a bitopological spaces, whereas the concept of quasi components in bitopological spaces was introduced by Reilly and Young [25]. The generalized closed set has been first studied and initiated by N.Levine in the year 1970 [19]. This generalized closed set has lead to significant contributions to generalization of continuity. Bhattacharya and Lahiri [4] introduced a closed set namely sg - closed set. Further Arya and Nour [1] defined generalized semi open sets ie) gs - open using semi open sets. Further Elvina Mary L and R.Myvizhi [13] has worked on *gs** - closed set. Recently, the notions of pairwise S*GO - connected spaces were introduced by K.Kannan [17] in bitopological spaces in 2009. In the year 2018, gs** - closed sets were introduced by D. Narasimhan and R.Subhaa [20].

Topology and Real Analysis is the study of variously modified forms of continuity, separation axioms, etc. by utilizing generalized open sets. One of the most well-known notions and also inspiration source are the notion of α-open [1] sets introduced by Njåstad in 1965. Since then, many mathematicians turned their attention to the generalization of various concepts in General Topology by considering α-open sets ([3], [22]) and generalized closed sets [2,10].In 1982 Dunham [14] used the generalized closed sets to define a new closure operator, and thus a new topology τ^*, on the space, and examined some of the properties of this new topology.

In this section, the new type of connected and disconnected spaces called pairwise DαO - connected spaces, pairwise DαO - disconnected spaces are introduced.

II. Preliminaries

Let (X,τ) or simply X denotes a topological space. For any subset $A \subseteq X$, the interior of A is the largest open set contained in A and the closure of A is the smallest closed set containing A and they are denoted by $int\,(A)$ and $cl\,(A)$ respectively.

Definition 2.1 A subset A of a topological space (X, τ) is called *a semi - open set* if $A \subseteq cl\,[int(A)]$.

Definition 2.2 A subset A of a topological space (X, τ) is called *a*

a. *generalized closed set* (*g* - closed set) if $cl\,(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
b. *gs closed set* (*g* - closed set) if $scl\,(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
c. *gs* - closed set* if $cl\,(A) \subseteq U$ whenever $A \subseteq U$ and U is gs - open in X.
d. *gs** - closed set* if $cl\,(A) \subseteq U$ whenever $A \subseteq U$ and U is gs* - open in X.
e. Dα- closed set [23] if $cl^*(int(cl^*(A))) \subseteq A$.

Definition 2.3 The intersection of all g - closed sets containing A [7] is called the g - closure of A and denoted by $cl^*(A)$, The union of all g – open sets contained in A [8] is called the g - interior of A and denoted by $int^*(A)$.

III. PAIRWISE DαO - CONNECTED SPACES

Let (X, τ_1, τ_2) or simply X denotes a bitopological space. For any subset $A \subseteq X$, τ_1 - Dα - *int (A)* , τ_1 - Dα - *cl (A)* denote Dα - *interior*, Dα - *closure* of a set A in X with respect to the topology τ_1, respectively. O.R.Sayed, A.M.Khalil [23] introduced the notion of Dα - closed sets in topological spaces in 2015. In this section, the new type of connected and disconnected spaces called pairwise Dα*O* - connected spaces, pairwise Dα*O* - disconnected spaces are introduced.

Definition 3.1. Let (X, τ_1, τ_2) be a bitopological space. Then (X, τ_1, τ_2) is said to be pairwise Dα*O* - connected if X cannot be expressed as the union of 2 non empty disjoint sets A and B such that $(A \cap \tau_1$ - Dα - $cl\ (B)) \cup (\tau_2$ - Dα - $cl\ (A) \cap B) = \phi$ … (1). If (1) is satisfied, we call A and B as pairwise Dα - separated sets. If $X = A \cup B$, where A, B satisfy (1), then X is called a pairwise Dα*O* - disconnected space. In this case, we write X = A / B a pairwise Dα *O* - separation of X.

Example 3.1. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\phi, X, \{a, b\}\}$.Then $X = \{b, c\} \cup \{a\}$ and $\{b, c\} \cap \{a\} = \phi$. Here $\{b, c\}$ and $\{a\}$ are τ_1 - Dα - open and τ_2 - Dα - open sets respectively. Therefore, (X, τ_1, τ_2) is pairwise Dα *O* - disconnected space.

Example 3.2. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$ and $\tau_2 = \{\phi, X, \{c\},\{b, c\}\}$. Let $G = \{a, c\}$ and $H = \{c\}$. Then $X \neq G \cup H$, where G is τ_1 - Dα - open and H is τ_2 - Dα - open, $G \cap H \neq \phi$. Hence (X, τ_1, τ_2) is pairwise Dα *O* - connected.

Theorem 3.1. If C is a pairwise Dα*O* - connected subset of a bitopological space (X, τ_1, τ_2) which has the pairwise Dα *O* - separation $X = A / B$ then $C \subset A$ or $C \subset B$.

Proof. Suppose that (X, τ_1, τ_2) has the pairwise Dα*O* - separation $X = A / B$. Then $X = A \cup B$, where A and B are nonempty disjoint sets such that $A \cap (\tau_1$ - Dα - $cl(B)) \cup (\tau_2$ - Dα - $cl(A) \cap B) = \phi$ … (1). Since $A \cap B = \phi$, we have $A = B^C$ and $B = A^C$… (2). Now, $((C \cap A) \cap \tau_1$ - Dα - $cl(C \cap B)) \cup (\tau_2$ - Dα - $cl(C \cap A) \cap (C \cap B)) \subseteq A \cap \tau_1$ - Dα - $cl(B)) \cup (\tau_2$ - Dα - $cl(A) \cap B) = \phi$ *[by (1)]*. $\Rightarrow C \cap A = \phi$ *or* $C \cap B = \phi$. $\Rightarrow C \subset A^C$ *(or)* $C \subset B^C$. $\Rightarrow C \subset B$ *(or)* $C \subset A$ [by (2)].

Theorem 3.2. If A is a pairwise Dα *O* - connected and A ⊆ B ⊆ τ_1 - Dα - cl(A) ∩ τ_2 - Dα - cl(A) then B is pairwise Dα*O* - connected.

Proof. Suppose that B is not pairwise Dα *O* - connected . Then $B = C \cup D$, where *C* and *D* are 2 non empty disjoint sets such that $(C \cap \tau_1$ - Dα - $cl(D)) \cup (\tau_2$ - Dα - $cl(C) \cap D) = \phi$. Since *A* is pairwise Dα*O* - connected, we have $A \subseteq C$ or $A \subseteq D$. Suppose $A \subseteq C$. Then $D \subseteq D \cap B \subseteq D \cap \tau_2$ - Dα - $cl(A) \subseteq D \cap \tau_2$ - Dα - $cl(C) = \phi$. Therefore, $\phi \subseteq D \subseteq \phi$. Consequently, $D = \phi$. Similarly, we can prove $C = \phi$ if $A \subseteq D$ {by theorem 3.1}. This is the contradiction to the fact that *C* and *D* are nonempty. Therefore, B is pairwise Dα*O* - connected.

Theorem 3.3. Let (X, τ_1, τ_2) be a bitopological space. If every 2 points of X are contained in some pairwise Dα*O* - connected space of X then X is pairwise Dα*O* - connected.

Proof. Assume that X is not pairwise Dα*O* - connected. Then $X = {}^{A}/_{B}$. $\Rightarrow X = A \cup B$, where *A* is τ_1 – Dα - open and τ_2 - Dα - open set with $A \cap B = \phi$. Let $x \in A$ and $y \in B$. By hypothesis, there exists a pairwise Dα - connected subset *C* of *X* such that $x \in C$ and $y \in C$, by the above theorem 3.2, $C \subset A$ or $C \subset B$. $\Rightarrow x, y \in A$ or $x, y \in B$, a contradiction. Hence X is pairwise Dα - connected.

IV. CONCLUSION

In this paper, the concept of pairwise Dα*O* - connected and pairwise Dα*O* – disconnected spaces are introduced and some of itscharacterizations are studied.

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